International Journal of Engineering Science Invention
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org Volume 2 Issue 7 ǁ July. 2013 ǁ PP.17-29

Entropy generation analysis for fully developed laminar convection in hexagonal duct subjected to constant heat flux and minimization of entropy generation by adjusting the shape of the cross section.

Dipak P. Saksena
Assistant Professor, Mechancial Engg. Dept.Institute of Diploma Studies.Nirmaunieversity

ABSTRACT: *The entropy generation of a fully developed laminar flow in a hexagonal duct is investigated in this study. A constant heat flux condition was applied in this analysis. Two fluids, water and engine oil, were used to study the effect of fluid properties on the entropy generation. The fluid properties were evaluated using average temperature between inlet and outlet duct sections. The aspect ratio of the hexagonal duct was varied to show its effect on the entropy generation. Attention was also given to the supplied heat flux affecting the entropy generation. Finally, the entropy generation calculated from the hexagonal duct was compared with that from rectangular and circular ducts having the same hydraulic diameter and cross sectional. Entropy generation in fully-developed flow through a duct with heat transfer is discussed. Methods are presented to minimize entropy generation by adjusting the shape of the duct's cross-section. Choosing a different cross-sectional shape allows for control of the competing fluid flow and heat transfer irreversibilities. By controlling the competing irreversibilities, the total entropy generation rate can be minimized. Given the flow rate, heat transfer rate, available cross-sectional area, and the fluid properties, a general design correlation is presented that allows for a determination of the optimal shape of a duct. area.*

I. INTRODUCTION

The entropy generation of a fully developed laminar flow in a hexagonal duct is investigated discussed in this study.

A constant heat flux condition was applied in this analysis. Two fluids, water and engine oil, were used to study the effect of fluid properties on the entropy generation. The fluid properties were evaluated using average temperature between inlet and outlet duct sections. The aspect ratio of the hexagonal duct was varied to show its effect on the entropy generation.

In the present study, the entropy generation for fully developed laminar convection through a hexagonal duct with constant heat flux is investigated. Water and engine oil are the fluids used to observe the significance of fluid viscosity and fluid properties on the entropy generation. Effects of aspect ratio and heat flux on the entropy generation are studied. Based on the same hydraulic diameter and cross-sectional area, the entropy generation for the hexagonal duct is compared to those for rectangular and circular ducts.

Forced convection heat transfer in a flow passage is affected by two types of losses, namely, loss associated with heat transfer through a temperature difference and loss associated with fluid friction. Entropy generation minimization has been proposed as a criterion for the design of flow passages in internal flow forced convection heat transfer configurations. Because entropy is generated by friction encountered in flowing fluids and by heat transfer through a temperature difference, a calculation of the overall entropy generation allows for an evaluation of these losses on a common scale. Moreover, because the entropy generation is a direct measure of the irreversibilities associated with heat transfer and fluid friction, the overall performance of a device containing heat transfer passages can be improved by calculating and minimizing the total entropy generation of the convective heat transfer process. A few past studies have attempted to compare the entropy generation in ducts with different cross-sectional shapes and to determine the cross-sectional shape that will yield minimum entropy generation [10–12]. Sahin finds that for high Reynolds number flows where fluid friction irreversibility dominates, the optimal shape for a flow channel is the circular shape in both laminar flow with a constant wall temperature [10] and in turbulent flow with constant wall heat flux [11].

II. ENTROPY GENERATION IN STEADY STATE FLOW THROUGH DUCTS.

In this section, the formulation of entropy generation equation is derived. All thermal properties are assumed to be uniform along the duct. The development of the entropy generation equation can be started by considering the fluid, which flows through a constant cross-sectional area duct, subjected to constant heat flux, as shown in Fig. 1. From the energy balance of the control volume of length dx, the heat transfer rate to the fluid inside the duct is:

Nomenclature for entropy generation

b, c, d duct sides, m
c_p specific heat, J/kg K
Dh hydraulic diameter, m
f friction factor or Darcy friction factor
h heat transfer coefficient, W/m2 K
k thermal conductivity, W/m K
L duct length, m
m mass flow rate, kg/s
Nu Nusselt number
p pressure, Pa
P perimeter, m
q" heat flux, W/m2
Q heat rate, W
Re Reynolds number
s specific entropy, J/kg$K
S entropy rate, W/K
St Stanton number

Greek symbols

ΔT wall-bulk fluid temperature difference (T_w-T)
γ aspect ratio of hexagonal duct
dimensionless entropy generation
θ hexagonal duct angle, degree
ρ fluid density, kg/m3
τ dimensionless temperature difference (τ=ΔT/T)
μ absolute viscosity, kg/m s

Superscripts

¯ quantity per unit mole

Subscripts

gen generation
i inlet
o outlet
w wall

Nomenclatures for Entropy generation minimization

A cross-sectional area
Bo duty parameter
C_f coefficient in friction factor
C_h coefficient in Nusselt number
c_p specific heat
D_h hydraulic diameter
$\frac{dT}{dx}$ axial temperature gradient
f Darcy friction factor
k thermal conductivity
m mass flow rate
Nu Nusselt number
P perimeter
P_r Prandtl number
q' heat transfer rate per unit length
R_A area based Reynolds number
Re Reynolds number
S_{gen} entropy generation per unit length
T temperature

Greek

α exponent in Nusselt number
β exponent in Nusselt number
γ exponent in friction factor
μ viscosity
ρ density
ϕ irreversibility distribution ratio
χ shape factor

Subscripts

opt at the optimum
min minimum value

Entropy generation minimization by adjusting the shape of the cross section:

III. ENTROPY GENERATION IN STEADY-STATE FLOW THROUGH DUCTS:

Consider the general internal flow configuration shown in Fig. 1. Fluid flows through a duct with a cross-sectional area A, a perimeter P, and a hydraulic diameter Dh = 4A/P. The shape of the crosssection is arbitrary but constant over the entire length of duct. A single-phase, incompressible and Newtonian fluid flows through the duct with a mass flow rate m at a bulk temperature T. Heat is transferred to the duct at a rate (per unit length) of q' , through the duct wall to the fluid across a temperature difference ΔT. Following Bejan [2], for T<<T, the entropy generation rate per unit length is given by

$$\dot{S}'_{gen} = \frac{\dot{q}'^2 D_h^2}{4NukAT^2} + \frac{1}{2}\frac{f\dot{m}^3}{\rho^2 A^2 T D_h}, \qquad (1)$$

where *Nu*, *f*, q', and k are the Nusselt number, the Darcy friction factor, the fluid density, and the fluid thermal conductivity, respectively.

$$Nu = C_h Re^{\alpha} P_r^{\beta} \qquad (2)$$

$$f = C_f Re^{-\gamma} \qquad (3)$$

Using the same notation as Ratts and Raut [4], the Nusselt number and friction factor for fully-developed laminar or turbulent flow are generalized as

where Re=m*Dh/*Aμ is the Reynolds number, Pr is the Prandtl number, and μ is the viscosity. The parameters C_h, *Cf*, α, β, and γ are tabulated in Table 1 for the circular cross-section and for rectangular and elliptical cross-sections with varying aspect ratios [13]. Additionally, in Table 1 the shape factor, defined as χ= P/Dh or $\chi=P^{2/}4A = 4A/Dh^2$, is shown for each cross-section. The shape factor is used throughout the following analysis and in the interpretation of results.
After substitution of Eqs. (2) and (3) into Eq. (1), the entropy generation rate is given by

$$\dot{S}'_{gen} = \frac{\dot{q}'^2 D_h^2}{4C_h kAT^2 Re^{\alpha} Pr^{\beta}} + \frac{1}{2}\frac{C_f\dot{m}^3}{\rho^2 A^2 T D_h Re^{\gamma}}. \qquad (4)$$

IV. DUCTS WITH SPECIFIED CROSS-SECTIONAL SHAPE

First, consider the entropy generation in a duct while holding constant the flow rate, the heat transfer rate, and the fluid properties. Assume that the channel has a specified cross-sectional shape; that is, χ is specified. Entropy generation can then be minimized by choosing the optimum cross-sectional size for the duct. For any duct with a specified shape factor, the size is determined
by either the hydraulic diameter or the cross-sectional area, since these parameters are related through the shape factor $\chi = 4A/Dh^2$
Furthermore, the definition of the shape factor is used to write.

$$Re = 4m/\chi \mu Dh \qquad (5)$$

For a constant flow rate, fluid properties, and cross-sectional shape, Eq. (5) shows that the Reynolds number can only be varied by changing the size of the cross-section (through changes in the hydraulic diameter). Thus, the optimum cross-sectional size can be found by determining the Reynolds number that minimizes entropy generation. Using the definitions for the shape factor and Reynolds number, and substituting into Eq. (4),

$$\dot{S}'_{gen} = \frac{\dot{q}'^2}{\chi C_h kT^2 Pr^{\beta}} Re^{-\alpha} + \frac{1}{128}\frac{C_f \chi^3 \mu^5}{\rho^2 T\dot{m}^2} Re^{5-\gamma}. \qquad (6)$$

The only parameter not held constant in the above equation is the Reynolds number. Differentiating and setting the result equal to
zero, one finds that the optimum Reynolds number is given by

$$\mathrm{Re}_{opt} = \left[\frac{128\alpha}{\chi^4(5-\gamma)C_f C_h}\right]^{\frac{1}{(5+\alpha-\gamma)}} Bo^{\frac{2}{(5+\alpha-\gamma)}} \mathrm{Pr}^{\frac{-\beta}{(5+\alpha-\gamma)}}, \tag{7}$$

where

$$Bo = \frac{\dot{q}'\rho\dot{m}}{\mu^{5/2}(kT)^{1/2}} \tag{8}$$

is the duty parameter. From the optimum Reynolds number, the optimum irreversibility distribution ratio (entropy generation due to fluid frictional losses divided by entropy generated by heat transfer through a temperature difference, or, the last term in Eq. (6) divided by the first term in Eq. (6) evaluated at Reopt) is evaluated as

$$\phi_{opt} = \frac{\alpha}{5-\gamma}. \tag{9}$$

Finally, the departure of the entropy generation from the optimum is evaluated as,

$$\frac{\dot{S}'_{gen}}{\dot{S}'_{gen,\min}} = \frac{(5-\gamma)}{(5+\alpha-\gamma)}\left(\frac{\mathrm{Re}}{\mathrm{Re}_{opt}}\right)^{-\alpha} + \frac{\alpha}{(5+\alpha-\gamma)}\left(\frac{\mathrm{Re}}{\mathrm{Re}_{opt}}\right)^{5-\gamma}, \tag{10}$$

where $S'_{gen,min}$ is the minimum entropy generation rate evaluated when Re = Re_{opt}.

Once the shape of the cross-section (the shape factor) is chosen, the generalized results above can be used to find the optimum Reynolds number (or equivalently, the optimum size of a cross-section), the optimum irreversibility distribution ratio, and the departure of the entropy generation from the optimum, provided that the friction factor and Nusselt number correlations are known and can be described by Eqs. (2) and (3).

V. DUCTS WITH A FIXED CROSS-SECTIONAL AREA

Now consider a similar situation, where the entropy generation is minimized in a duct flow with constant flow rate, constant heat transfer rate per unit length, and while holding the fluid properties constant. Consider a channel with a specified cross-sectional area, A, and minimize entropy generation by choosing the optimal crosssectional shape or shape parameter, χ , for the duct.

Once again, using the definitions of the shape factor and the Reynolds number, Eq. (4) can be arranged to read

$$\dot{S}'_{gen} = \frac{\dot{q}'^2 A\mu^2}{4C_h\dot{m}^2 kT^2\mathrm{Pr}^{\beta}}\mathrm{Re}^{2-\alpha} + \frac{1}{2}\frac{C_f\dot{m}^4}{\rho^2A^3\mu T}\mathrm{Re}^{-(\gamma+1)}. \tag{11}$$

Furthermore, the Reynolds number is

$$Re = \sqrt{\frac{4}{\chi A}\frac{\dot{m}}{\mu}}. \tag{12}$$

Holding constant the fluid properties, the heat transfer rate, the flow rate, for a duct with a fixed cross-sectional area, and assuming that the coefficients and exponents in the Nusselt number and friction factor correlations are only weakly dependent on the shape of the cross-section, the only variable in Eq. (11) is the Reynolds number. The Reynolds number can only vary with variations in the shape of the cross-section. To determine the optimal Reynolds number, or equivalently, the optimal shape for a cross-section, differentiate with respect to the Reynolds number and set the result equal to zero. This gives

$$\mathrm{Re}_{opt} = \left[\frac{(2-\alpha)}{2(\gamma+1)C_f C_h}\right]^{\frac{1}{(\alpha-\gamma-3)}} Bo^{\frac{2}{(\alpha-\gamma-3)}} R_A^{\frac{-8}{(\alpha-\gamma-3)}} \mathrm{Pr}^{\frac{-\beta}{(\alpha-\gamma-3)}}, \tag{13}$$

where R_A is a Reynolds number based on the area of the cross-section, defined by

$$R_A = \frac{\dot{m}}{\mu A^{1/2}}. \tag{14}$$

An optimal shape factor for a given flow configuration is evaluated from Eqs. (12) and (13) as

$$\chi_{opt} = 4\left[\frac{(2-\alpha)}{2(\gamma+1)C_f C_h}\right]^{\frac{-2}{(\alpha-\gamma-3)}} Bo^{\frac{-4}{(\alpha-\gamma-3)}} R_A^{\left[\frac{16}{(\alpha-\gamma-3)}+2\right]} \Pr^{\frac{2\beta}{(\alpha-\gamma-3)}}. \tag{15}$$

Finally, while holding the cross-sectional area constant, the optimum irreversibility distribution ratio and the departure of entropy generation from the minimum are given by

$$\phi_{opt} = \frac{(2-\alpha)}{(\gamma+1)} \tag{16}$$

$$\frac{\dot{S}'_{gen}}{\dot{S}'_{gen,\min}} = \frac{(\gamma+1)}{(3+\gamma-\alpha)}\left(\frac{\mathrm{Re}}{\mathrm{Re}_{opt}}\right)^{2-\alpha} + \frac{(2-\alpha)}{(3+\gamma-\alpha)}\left(\frac{\mathrm{Re}}{\mathrm{Re}_{opt}}\right)^{-(\gamma+1)}. \tag{17}$$

VI. RESULTS AND DISCUSSION

In this section, the entropy generation due to heat transfer and viscous friction is investigated for fully developed laminar flow in the hexagonal duct with constant heat flux forwater and engine oil. Some parameter values, employed in the calculations for both fluids, are shown in Table 1.

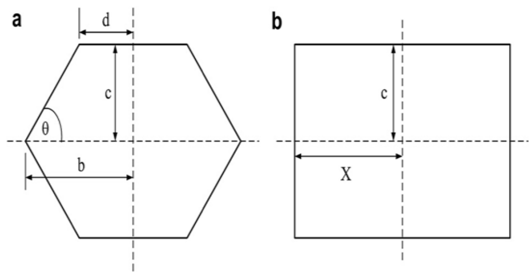

Fig. 2. Relating variables of (a) hexagonal and (b) rectangular ducts.

Table 1
Parameter values used in the analysis.

Parameter	Value
c (m)	0.04
θ (degree)	75.0
T_i (K)	298.0
L (m)	1.0
$\dot{m}$ (kg/s)	0.05
$\dot{q}''$ (W/m^2)	2000

In the analysis, the calculations started with finding the fluid outlet temperature, in which the constant specific heat of the fluid was initially assumed (for example, c_p = 4186 J/kgK for water). The average of the mean fluid temperature was found based on the initial specific heat. Next, the new specific heat value was determined based on the calculated mean temperature. The new outlet temperature of the fluid was calculated again using new specific heat value. The iterative procedure was continued until the difference of the average mean temperature taken from the last two iterations was less than 0.1 K. The specific heat and outlet temperature in the last iteration were adopted. For fully developed laminar flow through the hexagonal duct with constant heat flux, the Nusselt number and the fanning friction factor (the friction factor is four times greater than the fanning friction factor) are available in [18]. According to the reference, at a specific q (see Fig. 2a), both parameters are dependent on the aspect ratio, g. Finally Eq. (12) was used to calculate the entropy generation. If the dimensionless of the entropy generation is preferred, Eq. (13) can be employed.

6.1. The comparison of the entropy generation between water and engine oil

Table 2 The comparison of entropy generation components for water and engine oil at γ = 1.5.

Fluid	$\dot{S}_{gen,\Delta T}$	$\dot{S}_{gen,\Delta P}$	ψ
Water	2.90×10^{-1}	2.86×10^{-8}	1.39×10^{-3}
Engine oil	1.21	1.51×10^{-5}	3.10×10^{-2}

Table 2 shows the comparison of the entropy generation due to heat transfer and viscous friction for water and engine oil at γ =1.5.

Moreover, the dimensionless entropy generation is also expressed in the last column. From Table 2, it can be observed that the entropy generation due to heat transfer contributes to almost 100% in the total entropy generation. Similar result was also observed by Ben-Mansour and Sahin [19] for a circular pipe. When the results from both fluids are compared, the total entropy generation for the case of engine oil is higher than that for the case of water. Considering only the entropy generation due to heat transfer for engine oil, it is higher than that for water. Similarly, the entropy generation due to viscous friction for engine oil is higher than that for water. To investigate the reason, Eq. (12) has to be carefully considered. Although the same heat flux was supplied to both fluids, the outlet temperature for water and engine oil was different because of the difference in the specific heats. As mentioned, at a specific q, the Nusselt number is a function of the aspect ratio. Therefore, the same duct geometry for both fluids results in the same Nusselt number. Engine oil has lower specific heat value than that of water.

Therefore, with the same heat flux and inlet temperature, the outlet temperature of engine oil is higher than that of water. However, the thermal conductivity of engine oil is lower than that of water. The multiplication result leads to a lower denominator of the first term in Eq. (12) for the case of engine oil, compared with that for the case of water. As a result, the entropy generation due to heat transfer in the case of engine oil is higher than that in the case of water. Considering the entropy generation due to viscous friction, high viscosity in the case of engine oil causes this entropy generation to dramatically increase. The entropy generation due to viscous friction for the engine oil is about 530 times that for water, while the viscosity of engine oil is 424 times that of water. The total entropy generation for water and engine oil, calculated according to the above condition, are 2.90×10^{-1} W/K and 1.21 W/K, respectively. When these values are converted to dimensionless entropy generation, these become 1.39×10^{-3} and 3.10×10^{-2}, respectively.

6.2: The effect of duct aspect ratio and supplied heat flux

The aspect ratio of the duct was varied from 0.25 to 3.50 to study its effect on the entropy generation. The parameter *c* was fixed at 0.04 m. Since the duct aspect ratio changed, the parameter *b* also changed. The parameter d was computed using Eq. (16) with a fixed θ =75º and varied value of the aspect ratio. The values of the parameters, shown in Table 1, were used again in these calculations.

Fig. 3 shows the dimensionless entropy generation forwater and engine oil. It can be observed that the entropy generations for both fluids rapidly decrease when the aspect ratio increases. However, when the aspect ratio is higher than 2.0, the dimensionless entropy generations for both fluids are nearly constant. The reason is that when the parameter cis fixed, increasing the aspect ratio reduces the perimeter and hydraulic diameter. Consequently, the surface area of the duct is decreased. Even though the heat flux supplied to the fluid is constant, the total heat rate reduces because of a decrease in the heat transfer area. From Eq. (11), reducing the total heat rate significantly causes a decrease in the total entropy generation. If Eq. (12) is considered, the same result (decreasing the entropy generation) is caused by the decreasing duct perimeter and hydraulic diameter. However, the above explanation should be considered with the fact that the entropy generation due to viscous friction has insignificant effect on the total entropy generation, as previously discussed. For an aspect ratio higher than 2.0, increasing the aspect ratio results in a slowly decreasing duct perimeter. Therefore, the total heat rate slightly decreases and this event results the small variation in the total entropy generation. Moreover, if the result presented in Fig. 3 is carefully considered, it can be noticed that the dimensionless entropy generation for each fluid has a minimum value. The minimum dimensionless entropy generation for the case of water and the case of engine oil takes place at the aspect ratio of 2.642 and 2.627, respectively. The effect of heat flux on the entropy generation was also investigated.Water is the fluid used for this study. The parameters, given in Table 1 were again utilized, except for the heat flux. The

heat flux was varied from 1000 W/m^2 to 3600 W/m^2 with an increment of 200 W/m^2. The change in the dimensionless entropy generation was observed. Fig. 4a presents the result for the aspect ratio less than 2.642 (the minimum point), while Fig. 4b shows the result for the aspect ratio higher than 2.642. The figures clearly show the existence of a minimum point. For Fig. 4a, at a specific heat flux when the aspect ratio increases, the dimensionless entropy generation decreases. However, when the aspect ratio is higher than 2.642, the value of dimensionless entropy generation increases with increasing aspect ratio, as shown in Fig. 4b. For the effect of heat flux on the dimensionless entropy generation, both figures obviously illustrate that the heat flux has a strong influence on the entropy generation. At a high value of heat flux, the dimensionless entropy generation seems to linearly increase when heat flux increases.

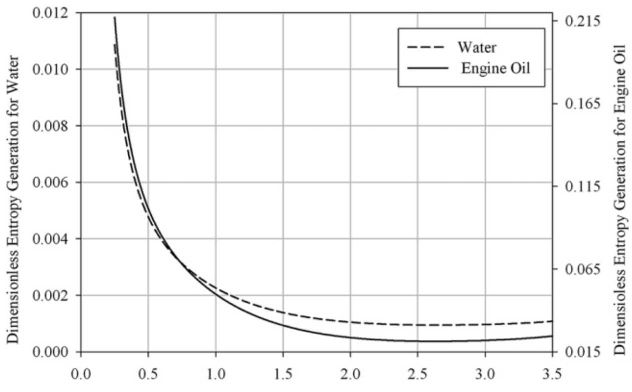

Fig.3. The dimensionless entropy generations for water and engine oil versus duct aspect ratio.

6.3: The comparison of the entropy generations for hexagonal duct and duct with different geometries:

In this section, the entropy generation for the hexagonal duct is compared with that for ducts having different geometries (rectangular and circular ducts). Two cases are considered. For the first case, the hydraulic diameter is the same for all duct geometrics. For the second case, the cross-sectional area is the same. The Nusselt numbers and the friction factors for the rectangular and circular ducts are available in [20].

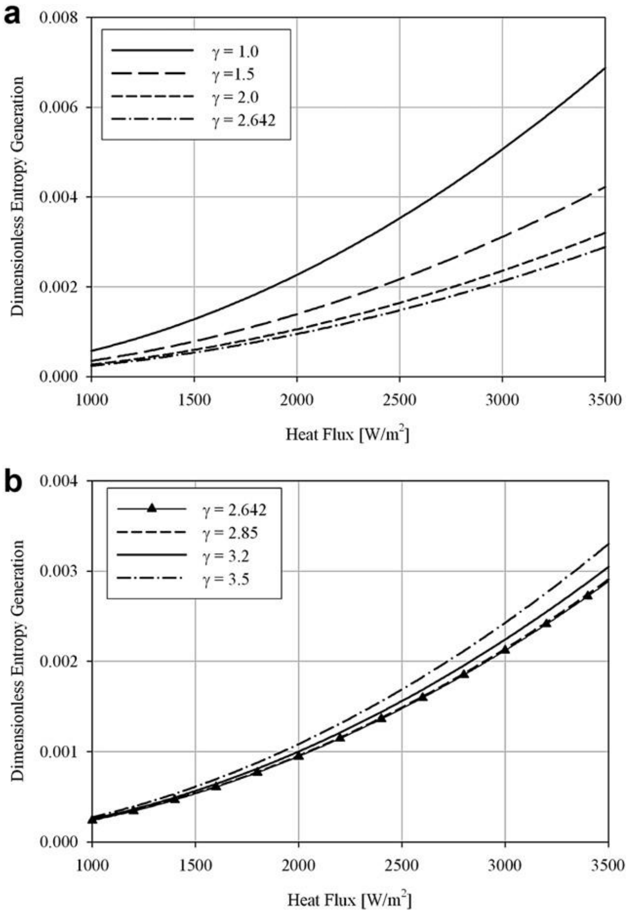

Fig. 4. The dimensionless entropy generation versus supplied heat flux, (a) for γ <=2:642, (b) for γ <= 2:642.

Considering the first case (the same hydraulic diameter) the parameters, shown in Table 1, were used again in the calculation. The hydraulic diameter for different geometries was adjusted to make them have the same values. For example, for the rectangular duct, one side was fixed to be equal to 2c (see Fig. 2b). The other side (2X) was calculated according to the equivalent hydraulic diameter condition. In the case of circular duct, its diameter was set to have the same value as hydraulic diameter of hexagonal duct. The calculationwas carried out and the result is shown in Fig. 5. The figure shows that, with the same hydraulic diameter, the circular duct is the best for this comparison. The use of circular duct results in the lowest dimensionless entropy generation through this examination range.

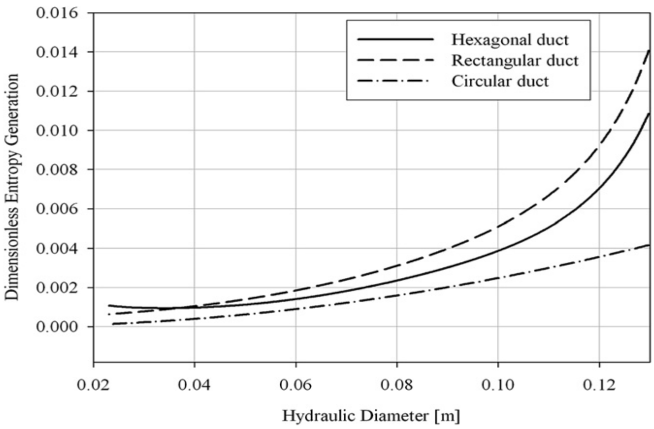

Fig. 5. The comparison of the dimensionless entropy generation for hexagonal, rectangular, and circular ducts with the equivalent hydraulic diameter condition.

For the hexagonal and rectangular ducts, the dimensionless entropy generations are not much different and they are slightly higher than that of circular duct for lower hydraulic diameter. However, when the hydraulic diameter increases, the dimensionless entropy generations for the hexagonal and rectangular ducts substantially increase and diverge from that of circular
duct.

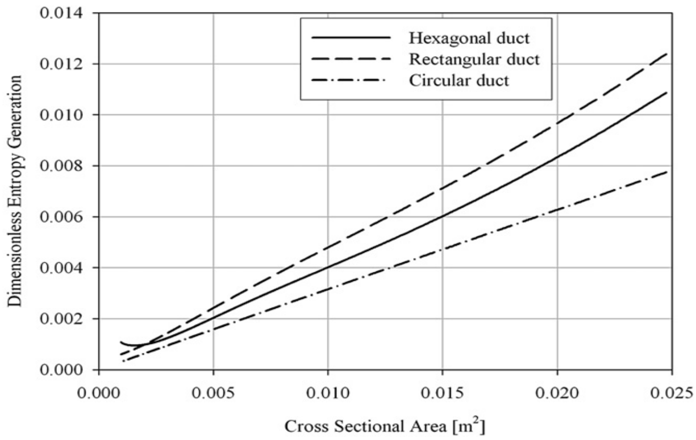

Fig. 6. The comparison of the dimensionless entropy generation for hexagonal, rectangular, and circular ducts with the equivalent cross-sectional area condition.

From the comparison, it can be implied that the circular duct is better than the rectangular and hexagonal ducts if the comparison is based on the entropy generation. The rectangular geometryis an inferior choice. This conclusion has also been mentioned in [6]. For the hexagonal duct, its dimensionless entropy generation is generally less than that of the rectangular. Nevertheless, for D_h < 0.036 m, the rectangular geometry is a little better than hexagonal geometry in the context of thermodynamic irreversibility. The circular duct gives the lowest dimensionless entropy generation. High thermodynamic irreversibility was observed from the case of the rectangular duct. For the hexagonal geometry, it has the dimensionless entropy generation higher than that of the circular geometry, but generally less than that of the rectangular one.

6.4 Ducts with specified cross-sectional shape:

Bejan determined the entropy generation, the irreversibility distribution ratio, and the departure of the entropy generation from the minimum for a circular cross-section in both laminar and turbulent flow [3]. In Bejan's analysis with the circular tube, the mass flow rate, the heat transfer rate per unit length, and the fluid properties were held constant and the tube diameter was adjusted to minimize entropy generation. Using the values from Table 1 in Eqs. (7) and (9) for the circular tube carrying laminar flow, one finds that $Re_{opt} = 0$ and that $\phi_{opt} = 0$. As suggested by Bejan, for a circular tube with laminar flow, the tube diameter should be large enough so that the entropy generation is dominated by the heat transfer contribution, which will result in a small value for the irreversibility distribution ratio, ϕ. As discussed above, this behavior for laminar tube flow is confirmed by the generalized expression in Eq. (9).

Using Table 1 and Eqs. (7), (9), and (10) for turbulent flow in a circular tube, the following are evaluated:

$$Re_{opt} = 2.023 Bo^{0.357} Pr^{-0.071} \quad (18)$$

$$\phi_{opt} = 0.167 \quad (19)$$

$$\frac{\dot{S}'_{gen}}{\dot{S}'_{gen,min}} = 0.857\left(\frac{Re}{Re_{opt}}\right)^{-0.8} + 0.143\left(\frac{Re}{Re_{opt}}\right)^{4.8}. \quad (20)$$

These equations are identical to those of Bejan. In both laminar and turbulent flow in circular tubes, the generalized results presented here reduce to expressions that are identical to the expressions derived by Bejan for circular tubes.

The generalized expressions in Eqs. (7), (9), and (10) can be used to determine the optimal size (to minimize entropy generation) of ducts with any cross-sectional shape, provided that the information contained in Table 1 are known for the flow through the particular cross-section chosen.

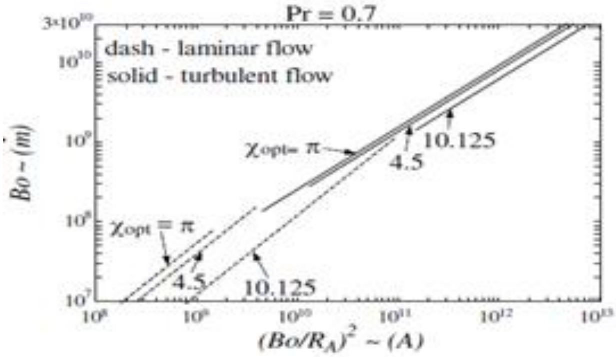

Fig.2. The shape factor that minimizes entropy generation for flow in a duct with specified flow rate, heat transfer rate, cross-sectional area and fluid properties. The figure is generated for circular, $\chi = \pi$, and rectangular $\chi = 4.5$ and $\chi = 10.125$, cross sections in laminar and turbulent flow. The figure is for fluids with Pr = 0.7.

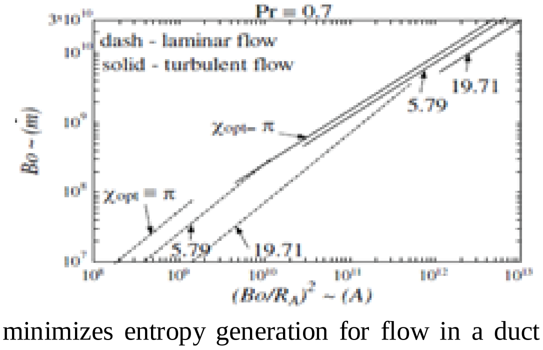

Fig. 3. The shape factor that minimizes entropy generation for flow in a duct with specified flow rate, heat transfer rate, cross-sectional area and fluid properties. The figure is generated for circular, $\chi = \pi$,, and elliptical, $\chi = 5.79$ and $\chi = 19.71$, cross sections in laminar and turbulent flow. The figure is for fluids with Pr = 0.7.

6.4: Ducts with a fixed cross-sectional area

For the case of internal convective heat transfer in a duct with a specified cross-sectional area, Eq. (15) is used to determine an optimal shape factor (a shape factor that minimizes entropy generation). To illustrate the determination of the optimal shape, results from Eq. (15) are presented graphically in Figs. 2 and 3.

Eq. (15) can be manipulated to give

$$\ln(Bo) = \left[\frac{5+\alpha-\gamma}{6+2\alpha-2\gamma}\right]\ln\left[\left(\frac{Bo}{R_A}\right)^2\right] + \left[\frac{\alpha-\gamma-3}{6+2\alpha-2\gamma}\right]\ln\left\{\left[\frac{(2-\alpha)}{2(\gamma+1)C_f C_h}\right]^{\frac{2}{(\alpha-\gamma-3)}}\frac{\chi_{opt}}{4\Pr^{2\beta/(\alpha-\gamma-3)}}\right\}. \quad (21)$$

As shown in Figs. 2 and 3, Eq. (15) results in straight lines on logarithmic scales when the parameters are plotted on the abscissa and ordinate, respectively.

$$\left(\frac{Bo}{R_A}\right)^2 = \frac{\dot{q}'^2\rho^2}{\mu^3 kT}A \quad \text{and} \quad Bo = \frac{\dot{q}'\rho}{\mu^{5/2}(kT)^{1/2}}\dot{m}, \quad (22)$$

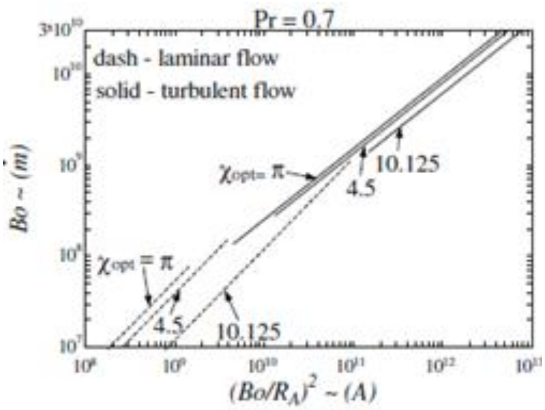

Fig. 2. *The shape factor that minimizes entropy generation for flow in a duct with specified flow rate, heat transfer rate, cross-sectional area and fluid properties. The figure is generated for circular, v = p, and rectangular, v = 4.5 and v = 10.125, crosssections in laminar and turbulent flow. The figure is for fluids with Pr = 0.7.*

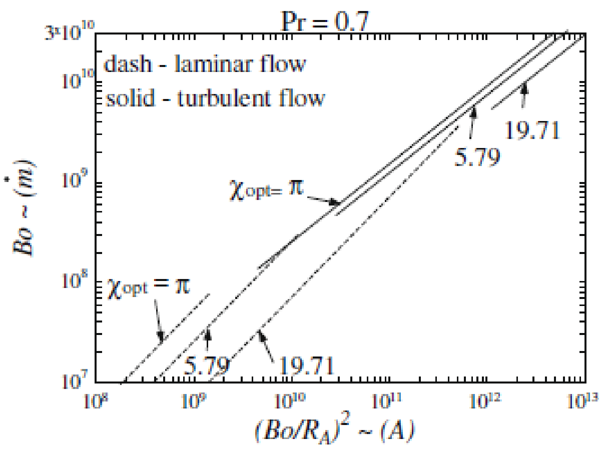

Fig. 3. The shape factor that minimizes entropy generation for flow in a duct with specified flow rate, heat transfer rate, cross-sectional area and fluid properties. The figure is generated for circular, v = p, and elliptical, v = 5.79 and v = 19.71, crosssections in laminar and turbulent flow. The figure is for fluids with Pr = 0.7.

Eq. (21) shows that, regardless of the shape of the cross-section, the slope of the lines on the plot will be constant and will be determined by the exponents in the Nusselt number and friction factor correlations (although the slopes for laminar and turbulent flow may be different from one another). Fig. 2 was generated using the exponents and coefficients for the Nusselt number and friction factor correlations in Table 1 for the circular tube and the rectangular cross-sections. Fig. 3 was generated using the values in Table 1 for the elliptical cross-sections. Although the lines in Figs. 2 and 3 have a constant slope regardless of the shape of the cross-section, both the shape factor and the value of the Prandtl number (Figs. 2) and 3 were generated for Pr = 0.7) of the fluid will shift the position of the line on the plane. Most importantly, if one considers a flow with a constant heat transfer rate per unit length and with constant fluid properties, the parameter on the horizontal axis scales directly with the cross-sectional area of the duct and the parameter on the vertical axis scales directly with the mass flow rate. For a particular convective flow with a given heat transfer rate, as soon as the flow rate and cross-sectional area are chosen, Figs. 2 and 3 can be used to determine the optimal shape factor. By way of example, consider a convective flow for which Pr = 0.7, $(Bo/R_A)^2 = 10^{12}$, and $Bo = 10^{10}$, and assume that we will use either a circular cross-section or a rectangular cross-section. In Fig. 2, this flow is represented by a point near the turbulent flow line of constant $\chi_{opt} = \pi$. These results would suggest that, for this particular flow, a circular cross-section (with $\chi_{opt} = \pi$) is the optimal cross-section to minimize entropy generation.

Now consider the same flow, however, double the available cross-sectional area. The flow parameters are now Pr = 0.7, $(Bo/R_A)^2 = 2 \times 10^{12}$, and $Bo = 10^{10}$. Notice that by increasing only the cross-sectional area we move horizontally to the right in Fig. 2 Eq. (22) shows that Bo remains constant after changing only the area). With an available cross-sectional area that is twice as large as in the previous example, the optimal shape factor is near $\chi_{opt} = 10.125$, suggesting that a rectangle with an aspect ratio of 8 is an optimal shape of the cross-section for the larger flow area duct rather than the circular cross-section. When the available cross-sectional area of the flow channel is doubled, the resistance to flow in the duct is reduced, thereby reducing the entropy generation associated with fluid friction. However, Eq. (16) indicates that in both examples above, the optimal irreversibility distribution ratio is

$$\phi_{opt} = \frac{(2-\alpha)}{(\gamma+1)} = \left(\frac{2-0.8}{0.2+1}\right) = 1, \qquad (23)$$

for these turbulent flows. To maintain this optimal irreversibility distribution ratio after an increase in the cross-sectional area (and the associated reduction in fluid friction), the entropy generation associated with heat transfer must also be reduced. The reduction in entropy generation associated with heat transfer is accomplished by choosing a cross-section with a larger perimeter compared to its cross-sectional area (a cross-section with a larger shape factor). Choosing a cross-sectional shape with a large perimeter increases the surface area available for heat transfer, reducing the entropy generation associated with heat transfer and restoring the balance required by Eq. (23) for minimum entropy generation. This example is continued by now considering a flow with half the available cross-sectional area as in the original example, so that Pr = 0.7, $(Bo/R_A)^2 = 5 \times 10^{11}$, and $Bo = 10^{10}$. Referring again to Fig. 2, we find ourselves at a point to the left of the constant $\chi_{opt} = \pi$ line. From Eq. (15), one calculates for these flow parameters, $\chi_{opt} = 0.48$. Because the circular cross-section has the smallest possible shape factor with $\chi_{opt} = \pi$, a circle would be used in this flow configuration to minimize entropy generation, although the true minimum could never be achieved. Figs. 2 and 3 show that flows with small flow rates generally require channels with larger shape factors to minimize entropy generation. For convective heat transfer with a low flow rate in a channel with a large cross-sectional area (the lower right portions of Figs. 2 and 3), the contribution to the total entropy generation by fluid friction is relatively low. In this situation, the total entropy generation is dominated by entropy generation due to heat transfer. To minimize entropy generation, the shape factor can be increased by introducing a geometry with more surface area available for heat transfer.

Next, consider an adiabatic flow , for which $\acute{q} = 0$. Circular or nearly circular cross-sections are used throughout engineered and natural systems to transport fluids while minimizing flow losses (flow resistance) in adiabatic flows [14]. In the case of an adiabatic flow, Bo = 0. In either laminar or turbulent flow, Eq. (15) suggests that $\chi_{opt} = 0$ for the adiabatic flow case. Again, because the smallest possible shape factor can be achieved with the circular cross-section, the entropy generation minimization analysis of Eq. (15) suggests that a circle is the most efficient cross-section for adiabatic flow.

Although Eq. (15) does reproduce the well-known result that the circular cross-section is the optimal shape for adiabatic flow through a duct, the entropy generation minimization presented here also provides a new result for ducts with forced convection heat transfer. The correlation in Eq. (15) and the examples shown above suggest that for flows dominated by heat transfer irreversibility (i.e., flows with large heat transfer rates, small flow rates, and large available cross-sectional area, or equivalently, flows with large Bo and small RA) the circular cross-section is not ideal and large aspect ratio channels should be used to minimize entropy generation.

VII. CONCLUSIONS

The entropy generation for fully developed laminar convection through a hexagonal duct with constant heat flux is investigated. The result shows that the fluid properties strongly affect the entropy generation. The total entropy generation, for the case of engine oil, is about 4.2 times of that for the case of water. The analysis indicated that when the aspect ratio increases, the dimensionless entropy generation decreases. This is because, at constant heat flux, when the parameter c is fixed and the aspect ratio increases, the total heat transfer rate decreases. Consequently, the entropy generation due to heat transfer reduces. However, for aspect ratio that are higher than 2.642 and 2.627 (for the case of water and engine oil, respectively), the dimensionless entropy generation slightly increases with increasing aspect ratio. The comparison, based on the same hydraulic diameter and crosssectional area conditions, illustrates that the circular duct has the lowest dimensionless entropy generation, while the rectangular geometry generally causes the highest dimensionless entropy generation. For the case of the hexagonal duct, its dimensionless entropy generation is usually in the middle. Entropy generation in fully-developed convective heat transfer has been investigated. Generalized correlations to determine the optimum cross-sectional shape of a flow passage to minimize entropy production have been presented. The equations confirm the well-known conclusion that in adiabatic flow, the circular crosssection will minimize flow resistance, which is reflected by a minimization of the entropy generation. However, in flows with heat transfer, the correlations developed suggest that the circular cross-section may not always be ideal. In situations where the heat transfer irreversibility dominates (with low flow rates, large available cross-sectional areas, and high heat transfer rates), a duct with a large wetted perimeter (for example, a rectangular channel with a large aspect ratio) will increase the surface area available for heat transfer and will minimize the overall entropy generation.

REFERENCES

[1]. Bejan A. A study of entropy generation in fundamental convective heat transfer. Journal of Heat Transfer 1979;101(4):718e25.
[2]. Bejan A. Entropy generation through heat and fluid flow. New York: Wiley; 1982.
[3]. Bejan A. Entropy generation minimization. Boca Raton, FL: CRC Press; 1996.
[4]. Bejan A. Entropy generation minimization: the new thermodynamics of finitesize devices and finite-time processes. Journal of Applied Physics 1996;79 (3):1191e218.
[5]. Sahin AZ. Thermodynamics of laminar viscous flow through a duct subjected to constant heat flux. Energy 1996;21(12):1179e87.
[6]. Sahin AZ. Irreversibilities in various duct geometries with constant wall heat flux and laminar flow. Energy 1998;23(6):465e73.
[7]. Sahin AZ. Entropy generation and pumping power in a turbulent fluid flow through a smooth pipe subjected to constant heat flux. Exergy 2002;2 (4):314e21.
[8]. Ko TH, Ting K. Entropy generation and optimal analysis for laminar forced convection in curved rectangular ducts: a numerical study. International Journal of Thermal Sciences 2006;45(2):138e50.
[9]. Ko TH, Ting K. Entropy generation and thermodynamic optimization of fully developed laminar convection in a helical coil. International Communications in Heat and Mass Transfer 2005;32(1e2):214e23.
[10]. Ko TH, Ting K. Optimal Reynolds number for the fully developed laminar forced convection in a helical coiled tube. Energy 2006;31(12):2142e52.
[11]. Satapathy AK. Thermodynamic optimization of a coiled tube heat exchanger under constant wall heat flux condition. Energy 2009;34(9):1122e6.
[12]. McCutcheon SC, Martin JL, Barnwell TO, Water quality. In: Maidment DR, editor. Handbook of hydrology. New York: McGraw-Hill. p. 11.1e11.73.
[13]. White FM. Viscous fluid flow. New York: McGraw-Hill; 1974.
[14]. Reid RC, Prausnitz JM, Poling BE. The properties of gases and liquids. New York: Prentice Hall; 1987.
[15]. McBride BJ, Gordon S, Reno MA. Coefficients for calculating thermodynamic and transport properties of individual species. NASA Technical Memorandum 4513, NASA; 1993.
[16]. Incropera FP, Dewitt DP, Bergman TL, Lavine AS. Fundamentals of heat and mass transfer. 6th ed. New York: John Wiley and Sons; 2007.
[17]. Liley PE. In: Rohsenow W, Hartnett JP, Ganic EN, editors. Handbook of heat transfer fundamentals. 2nd ed. New York: McGraw-Hill. 3.1e3.135.
[18]. Sadasivam R, Manglik RM, Jog MA. Fully developed forced convection through tranpezoidal and hexagonal ducts. International Journal of Heat and Mass Transfer 1999;42(23):4321e31.
[19]. Ben-Mansour R, Sahin AZ. Entropy generation in developing laminar fluid flow through a circular pipe with variable properties. Heat and Mass Transfer 2005;42(1):1e11.
[20]. Ebadian MA and Dong ZF. In: Rohsenow W, Hartnett JP, Cho YI, editors. Handbook of heat transfer fundamentals, 3rd ed. New York: McGraw-Hill. 5.1e5.137.
[21]. Bejan, General criterion for rating heat exchanger performance, Int. J. Heat Mass Transfer 21 (1978) 655–658.
[22]. Bejan, Entropy Generation through Heat and Fluid Flow, John Wiley & Sons, New York, 1982. pp. 105–109.
[23]. A. Bejan, A study of entropy generation in fundamental convective heat transfer, ASME J. Heat Transfer 101 (1979) 718–725.
[24]. E.B. Ratts, A.G. Raut, Entropy generation minimization of fully developed internal flow with constant heat flux, ASME J. Heat Transfer 126 (2004) 656– 659.
[25]. H.F. Oztop, Effective parameters on second law analysis for semicircular ducts in laminar flow and constant wall heat flux, Int. Commun. Heat Mass Transfer 32 (2005) 266–274.

[26]. D.H. Richardson, D.P. Sekulic, A. Campo, Low Reynolds number flow inside straight micro channels with irregular cross-sections, Heat Mass Transfer 36 (2000) 187–193.
[27]. A.Z. Sahin, Thermodynamics of laminar viscous flow through a duct subjected to constant heat flux, Energy 21 (1996) 1179–1187.
[28]. A.Z. Sahin, Entropy generation in turbulent liquid flow through a smooth duct subjected to constant wall temperature, Int. J. Heat Mass Transfer 43 (2000) 1469–1478.
[29]. S.Y. Wu, Y.R. Li, Y. Chen, L. Xiao, Energy transfer characteristics of forced convective heat transfer through a duct with constant wall temperature, Energy 32 (2007) 2385–2395.
[30]. A.Z. Sahin, A second law comparison for optimum shape of duct subjected to constant wall temperature and laminar flow, Heat Mass Transfer 33 (1998) 425–430
[31]. A.Z. Sahin, Irreversibilities in various duct geometries with constant wall heat flux and laminar flow, Energy 23 (1997) 465–473.
[32]. D.P. Sekulic, A. Campo, J.C. Morales, Irreversibility phenomena associated with heat transfer and fluid friction in laminar flows through singly connected ducts, Int. J. Heat Mass Transfer 40 (1997) 905–914.
[33]. Y.A. Cengel, Heat Transfer: A Practical Approach, McGraw-Hill, Boston, 1998. p. 380.
[34]. A. Bejan, Shape and Structure, from Engineering to Nature, Cambridge University Press, Cambridge, 2000. pp. 127–128.
[35]. Sompop Jarungthammachote* Faculty of Engineering at Si Racha, Kasetsart University Si Racha Campus, 199 Sukhumvit Road, Si Racha, Chonburi, 20230,2010. ThailandEntropy generation analysis for fully developed laminar convection in hexagonal duct subjected to constant heat flux. Energy 35 (2010) 5374-5379
[36]. Todd A. Jankowski * Mechanical and Thermal Engineering Group (AET-1), Los Alamos National Laboratory, MS J580, Los Alamos, NM 87545, USA. Minimizing entropy generation in internal flows by adjusting the shape of the cross-section.